2025/FYUG/ODD/SEM/
MTMDSC–301T/072

FYUG Odd Semester Exam., 2025

MATHEMATICS

(5th Semester)

Course No. : MTMDSC–301T

(Partial Differential Equations)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) Eliminate the arbitrary constants a and b from $z=(x-a)^2+(y-b)^2$ to form a partial differential equation.

(b) Form a partial differential equation by eliminating the arbitrary function f from the relation $z=x+y+f(xy)$.

(c) Solve the equation $\dfrac{\partial^2 z}{\partial x \partial y}=0$.

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2. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z,\ x^2+y^2-z^2)=0$. 5

(b) Find the integral surface of the linear partial differential equation

$$x(y^2+z)\frac{\partial z}{\partial x}-y(x^2+z)\frac{\partial z}{\partial y}=(x^2-y^2)z$$

which contains the straight line $x+y=0,\ z=1$. 5

(c) Solve the PDE

$$\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial z}{\partial y}=1$$

subject to the conditions $u(x, 0)=0$, $\frac{\partial u}{\partial y}(0, y)=\sin y$. 5

(d) Solve : 5

$$(x^2+2y^2)p-xyq=xz$$

UNIT—II

3. Answer any *two* of the following questions : $2\times2=4$

(a) Write down the Charpit's auxiliary equations of $q=3p^2$.

(b) Find a complete integral of the PDE $z=pq$.

(c) Solve :

$$z=px+qy+pq$$

4. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Show that the partial differential equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution. 5

(b) Find a complete integral of the partial differential equation

$$(x^2-y^2)pq-xy(p^2-q^2)=1 \quad 5$$

(c) Find a complete and singular solution of the partial differential equation $(p^2+q^2)y=qz$. 5

(d) Solve : 5

$$z=px+qy+c\sqrt{1+p^2+q^2}$$

UNIT—III

5. Answer any *two* of the following questions : $2\times2=4$

(a) Solve :

$$x\frac{\partial^2 z}{\partial x^2}=\frac{\partial z}{\partial x}$$

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(b) Solve :

$$(D^2 + 2DD' + D'^2)z = 0$$

(c) Find the particular integral of the PDE

$$(D^2 + 3DD' + 2D'^2)z = x + y$$

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Solve : 5

$$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y)$$

(b) Solve : 2+3=5

(i) $(D^2 - D'^2 + D - D')z = 0$

(ii) $t + s + q = 0$

(c) Solve : 5

$$(D^2 - 2DD' - 15D'^2)z = 12xy$$

(d) Solve the Cauchy problem

$$\frac{\partial^2 z}{\partial y^2} - 2z = 0$$

$$u(x, 0) = x^3, \ \frac{\partial u}{\partial y}(x, 0) = e^{-x}$$ 5

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Describe the classification of the linear second order PDE

$$A\frac{\partial^2 z}{\partial x^2} + B\frac{\partial^2 z}{\partial x \partial y} + C\frac{\partial^2 z}{\partial y^2} + D\frac{\partial z}{\partial x} + E\frac{\partial z}{\partial y} + Fz = G$$

(b) Find the canonical form of the partial differential equation

$$2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} + 8z = 0$$

(c) Solve the PDE $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ using the method of separation of variables.

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Obtain the canonical form of the PDE

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + xyz = 1$$

and hence solve it. 5

(b) Solve the PDE

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$$

using the method of separation of variables. 5

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(c) Reduce the PDE

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

into canonical form and hence solve it. 5

(d) Solve the PDE

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

using the method of separation of variables. 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define homogeneous and non-homogeneous boundary conditions of a second-order PDE.

(b) Give physical interpretation of the variables x, t and u in the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(c) Find the eigenvalues of the differential equation

$$\frac{d^2 y}{dx^2} - \lambda^2 y = 0$$

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Derive heat conduction equation for a bar. 5

(b) Describe Fourier half-range sine and cosine series. 5

(c) Derive the one-dimensional homogeneous wave equation. 5

(d) Solve completely the heat conduction equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

subject to the initial condition

$$u(x, 0) = f(x),\ 0 < x < 1$$

and boundary conditions

$$u(0, t) = 0;\ u(1, t) = 0,\ t > 0$$ 5

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