FYUG Even Semester Exam., 2026

MATHEMATICS

(6th Semester)

Paper Code : MTMDSC 351T

(Complex Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Show that for any two complex numbers z and w, $|z-w| \geq |z|-|w|$.

(b) Prove that $\operatorname{Re}(z) > 0$ and $|z-1| < |z+1|$ are equivalent.

(c) Find

$$\operatorname*{Lt}_{z \to e^{\pi i/3}} (z - e^{\pi i/3}) \frac{z}{z^3+1}$$

2. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Explain the geometrical interpretation of $\arg\left(\frac{z-\alpha}{z-\beta}\right)$. Hence find the condition for collinearity of three complex numbers z, α and β. 4+1=5

(b) Show that the equation of the straight line in the Argand plane can be put in the form $b\bar{z}+\bar{b}z=c$. 5

(c) Determine the region of the complex plane described by $|z+1|+|z-1|\le 4$. Illustrate the same with a diagram. 4+1=5

(d) Prove that a continuous function in a closed and bounded region in the Z plane is bounded. 5

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Write down the Cauchy-Riemann equations in polar form.

(b) Define harmonic function.

(c) Define analytic function with example.

4. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) If a function is differentiable in the Z plane, then prove that it is continuous in the Z plane. Give an example to show that the converse is not true. 3+2=5

(b) Show that the function

$$f(z)=e^{-z^{-4}},\quad z\neq 0$$
$$=0\quad,\quad z=0$$

is not analytic at $z=0$, although Cauchy-Riemann equations are satisfied at that point. 5

(c) Derive Cauchy-Riemann equations in Cartesian form. 5

(d) Prove that $e^x\cos y$ is harmonic. Determine its harmonic conjugate and find the corresponding analytic function in terms of z. 2+3=5

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Explain simply connected region and multiply connected region.

(b) Show that

$$\left| \int_{\Gamma} \frac{dz}{z^2+1} \right| \le \frac{\pi}{3}$$

where Γ denotes the circle $|z|=2$ in the 1st quadrant.

(c) State Morera's theorem.

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) State and prove Cauchy's theorem. 2+4=6

(b) Prove that if $f(z)$ is integrable along a curve C having finite length L and if there exists a positive number M such that $|f(z)| \le M$ on C, then

$$\left| \int_C f(z)\,dz \right| \le ML$$

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(c) Prove Cauchy-Goursat theorem for a triangle. 6

(d) Evaluate

$$\oint_C \frac{dz}{z-3}$$

where C is the circle $|z-2|=5$. Does the result contradict Cauchy's theorem? Justify. 3+1=4

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) State Cauchy's integral formula.

(b) State Taylor's theorem.

(c) State the condition of convergence of the series

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots$$

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Use Cauchy's integral formula to evaluate

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$$

where C is the circle $|z|=3$. 5

(b) State and prove Liouville's theorem. 1+4=5

(c) State the fundamental theorem of algebra and prove it by using Liouville's theorem. 1+4=5

(d) Expand the following function in a Taylor's series about $z=0$ and determine the region of convergence in each case : 2+3=5

(i) e^z

(ii) $\sin z$

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) Define pole with an example.

(b) Define Laurent's theorem.

(c) Define entire function with an example.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) State and prove Cauchy's residue theorem. 1+4=5

(b) Use calculus of residues to prove that

$$\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta = \frac{2\pi}{b^2}\left(a-\sqrt{a^2-b^2}\right)$$ 5

(c) Expand $\frac{e^z}{z^3}$ and $\frac{z^3}{e^z}$ in Laurent series about $z=0$. Hence identify the types of singularities in each case. 3+2=5

(d) Use calculus of residues to prove that

$$\int_0^{2\pi} e^{\cos\theta}\cdot\cos(n\theta-\sin\theta)\,d\theta = \frac{2\pi}{\underline{|n}}$$ 5

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