FYUG Even Semester Exam., 2026

MATHEMATICS

(6th Semester)

Paper Code : MTMDSC 354T

(Linear Programming)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions.

UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) What are the three main steps in formulating a linear programming model?

(b) Define a convex set in the Euclidean space $\mathbb{R}^n$ and give an example of a convex set in $\mathbb{R}^2$.

(c) Give an example of a boundary point of a convex set which is not an extreme point and also give an example of a convex set in which every boundary point is also an extreme point.

2. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Solve graphically the LPP : 5

Maximize $Z = 4x + y$

subject to the constraints

$$x + 2y \leq 3$$
$$4x + 3y = 6$$
$$3x + y \geq 3$$
and $x, y \geq 0$

(b) *(i)* Is the union of two convex sets in $\mathbb{R}^2$ is always a convex set? Justify your answer. 2

(ii) Examine whether

$x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 6$

is a basic feasible solution of the system of equations :

$$x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = x_4$$ 3

(c) *(i)* Write the following LPP in standard form : 3

Minimize $Z = 2x_1 + x_2 - 6x_3 - 4x_4$

subject to the constraints

$$3x_1 + x_4 \geq 25$$
$$x_1 + x_2 + x_3 + x_4 = 20$$
$$4x_1 + 6x_3 \leq 5$$
$$2 \leq x_1 + 3x_3 + 2x_4 \leq 30$$
and $x_1, x_2, x_3, x_4 \geq 0$

(ii) Find a basic feasible solution of the system of equations :

$$x_1 - x_2 + 2x_3 = 1$$
$$2x_1 - 2x_2 + 3x_3 = 2$$

How many basic solutions can this system have? 2

(d) A transport company has five offices in five localities *A, B, C, D, E*. Some day offices located at *A* and *B* had 8 and 10 spare trucks, whereas offices at *C, D, E* required 6, 8, 4 trucks respectively. The distances in kilometres between the five localities are given in the table below :

		To		
		C	*D*	*E*
From	*A*	2	5	3
	B	4	2	7

How should the trucks from *A* and *B* be sent to *C, D, E* so that the total distance covered by the trucks is minimum? Pose the problem as an LPP. 5

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) What is an artificial variable? Why is it called artificial?

(b) Write a short note on Big-M method.

(c) Construct the auxiliary LPP of two-phase method for the LPP

$$\text{Maximize } Z = 5x_1 + 8x_2$$

Subject to

$$3x_1 + 2x_2 \geq 3$$
$$x_1 + 4x_2 \geq 4$$
$$\text{and } x_1, x_2 \geq 0$$

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Solve the following LPP using Simplex method : 7

$$\text{Maximize } Z = 3x_1 + 5x_2 + 4x_3$$

Subject to the constraints

$$2x_1 + 3x_3 \leq 18$$
$$2x_2 + 5x_3 \leq 18$$
$$3x_1 + 2x_2 + 4x_3 \leq 25$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

(b) If the objective function of an LPP assumes its optimal value at more than one extreme points, then prove that every convex combination of these extreme points gives optimal value of the objective function. 3

(c) Solve the following LPP by Big-M method : 7

$$\text{Minimize } Z = x_1 + x_2 + x_3$$

Subject to the constraints

$$x_1 - 2x_2 \leq 3$$
$$2x_2 - x_3 \geq 4$$
$$x_1 - 3x_2 + 4x_3 = 5$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

(d) While solving an LPP by two-phase method, at the end of phase–I what are the three possible cases that may arise and what will be the conclusions? 3

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) What is duality? Write two advantages of duality.

(b) State the weak duality theorem.

(c) Find the dual of the LPP given below :

$$\text{Minimize } Z = 25x_1 + 50x_2$$

Subject to the constraints

$$3x_1 + x_2 \geq 8$$
$$4x_1 + 3x_2 \geq 9$$
$$x_1 + 3x_2 \geq 7$$

and $x_1, x_2 \geq 0$

6. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] : 10

(a) Prove that the dual of the dual of a given primal is the primal itself. 4

(b) Solve both the primal and the dual of the LPP : 6

$$\text{Minimize } Z = 3x_1 + 2x_2$$

Subject to the constraints

$$7x_1 + 2x_2 \geq 30$$
$$5x_1 + 4x_2 \geq 20$$
$$x_1 + 4x_2 \geq 8$$

and $x_1, x_2 \geq 0$

(c) If any variable of the primal problem be unrestricted in sign, then prove that the corresponding dual constraint will be an equality. 3

(d) Using duality, solve the following LPP : 7

$$\text{Maximize } Z = 3x_1 - 2x_2$$

Subject to the constraints

$$x_1 \leq 4,\ x_2 \leq 6,\ x_1 + x_2 \leq 5$$
$$-x_2 \leq -1 \text{ and } x_1, x_2 \geq 0$$

UNIT—IV

7. Answer any *two* of the following questions : $2 \times 2 = 4$

(a) In a balanced transportation problem with m origins and n destinations, what is the number of independent constraints? Also what is the number of variables?

(b) Define a loop in a transportation table. What is the utility of constructing a loop in a transportation table while solving for optimality?

(c) Find an initial basic feasible solution of the following transportation problem using North-West corner rule :

	D_1	D_2	D_3	D_4	$a_i \downarrow$
O_1	4	6	9	5	16
O_2	4	2	7	1	14
O_3	2	5	2	8	10
$b_j \rightarrow$	12	7	6	15	

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) State and prove a necessary and sufficient condition for a transportation problem to have a feasible solution. 5

(b) Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the corresponding transportation cost : 5

Destinations

		D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
Origins	O_1	2	11	10	3	7	4
	O_2	1	4	7	2	1	8
	O_3	3	9	4	8	12	9
	$b_j \rightarrow$	3	3	4	5	6	

(c) Explain the steps in finding the initial basic feasible solution of a balanced transportation problem by Vogel's approximation method. 4

(d) Find initial basic feasible solution of the following transportation problem by column minima method and matrix minima method and also compare the transportation costs : 6

Destinations

		D_1	D_2	D_3	D_4	$a_i \downarrow$
Origins	O_1	6	4	2	7	8
	O_2	5	1	4	6	14
	O_3	6	5	2	5	9
	O_4	4	3	2	1	11
	$b_j \rightarrow$	7	13	12	10	

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Write the mathematical formulation of an assignment problem as an LPP.

(b) Explain how we can resolve degeneracy in a transportation problem.

(c) Distinguish between a transportation problem and an assignment problem.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Find the optimal solution of the following transportation problem : 5

Destinations

	D_1	D_2	D_3	D_4	$a_i \downarrow$
Origins O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
$b_j \rightarrow$	20	40	30	10	

(b) Solve the following unbalanced assignment problem to minimize the assignment cost : 5

Persons

	P_1	P_2	P_3
Jobs I	11	17	8
II	9	7	12
III	13	16	15
IV	21	24	17

(c) Find initial basic feasible solution of the following transportation problem by matrix minima method and hence find the optimal solution by MODI method, resolving degeneracy if any : 6

Destinations

	D_1	D_2	D_3	D_4	*Supply* $\downarrow$
Sources S_1	1	7	3	4	30
S_2	4	1	2	5	20
S_3	6	1	7	4	30
Demand $\rightarrow$	30	10	20	40	

(d) Three persons are being considered for three open positions. Each person has been given a rating for each position as shown in the following table :

Positions

	I	II	III
Persons A	7	5	6
B	8	4	7
C	9	6	4

Assign each person to one and only one position in such a way that the sum of ratings for all the three persons is maximum. 4

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